

Scattering of a solitary pulse on a local defect or breather

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Abstract

A model is introduced to describe guided propagation of a linear or nonlinear pulse which encounters a localized nonlinear defect, that may be either static or breather-like one. The model with the static defect directly applies to an optical pulse in a long fiber link with an inserted additional section of a nonlinear fiber. A local breather which gives rise to the nonlinear defect affecting the propagation of a narrow optical pulse is possible in a molecular chain. In the case when the host waveguide is linear, the pulse has a Gaussian shape. In that case, an immediate result of its interaction with the nonlinear defect can be found in an exact analytical form, amounting to transformation of the incoming Gaussian into an infinite array of overlapping Gaussian pulses. Further evolution of the array in the linear host medium is found numerically by means of the Fourier transform. An important ingredient of the linear medium is the third-order dispersion, that eventually splits the array into individual pulses. If the host medium is nonlinear, the input pulse is naturally taken as a fundamental soliton. The soliton is found to be much more resistant to the action of the nonlinear defect than the Gaussian pulse in the linear host medium, for either relative sign of the bulk and local nonlinearities. In this case, the third-order-dispersion splits the soliton proper and wavepackets generated by the action of the defect.

I. INTRODUCTION

The interaction of traveling solitary pulses (which, in particular but not necessarily, may be solitons, that we realize here as pulses in nonlinear media maintaining a steady shape while propagating) with various local imperfections or pinned dynamical excitations is a problem of fundamental importance. For direct experimental observation, the most straightforward case is the propagation of a pulse in an optical fiber (linear or nonlinear), in which a strong localized nonlinear defect represents an inserted piece of a *dispersion-shifted* fiber (DSF) [1], that has finite nonlinearity and negligible dispersion. This configuration, which may be quite useful for optical telecommunications [1], is described by the following version of the nonlinear Schrödinger (NLS) equation,

$$iu_z - \frac{1}{2}\beta_2 u_{\tau\tau} + \frac{i}{6}\beta_3 u_{\tau\tau\tau} + \gamma |u|^2 u + \Gamma \delta(z) |u|^2 u = 0, \quad (1)$$

where $u(z, \tau)$ is a local amplitude of the electromagnetic wave, z is the propagation coordinate,

$$\tau \equiv t - z/V_0 \quad (2)$$

is the so-called reduced time, V_0 being the group velocity of the carrier wave, β_2 and β_3 are coefficients of the second-order and third-order group-velocity dispersion (GVD) in the fiber, and γ is the nonlinearity coefficient of the host (system) fiber. The strength of the localized nonlinear defect is $\Gamma \equiv \gamma_{\text{DSF}} L$, where γ_{DSF} and L are the nonlinearity coefficient and actual length of the above-mentioned finite-length DSF inserted into the host fiber at the point $z = 0$; this insertion may be represented by the delta-function in Eq. (1), as the DSF's dispersion is negligible [1]. Note that the nonlinear coefficient γ is always positive, as it is induced by the Kerr effect in the optical fiber, that always has the sign corresponding to self-focusing [2]. On the other hand, the GVD coefficient β_2 may be either positive or negative, which corresponds, respectively, to normal and anomalous dispersion [2]. The third-order dispersion (TOD) coefficient β_3 is frequently neglected, unless β_2 is very small or if the pulse is very narrow in the τ -domain. However, TOD will play an essential role in the present work. Note that β_3 is usually positive in optical fibers [2].

A qualitatively different version of the model containing the local defect pertains to the case when it describes the interaction of the propagating modes (pulses) with a localized defect in the form of a pinned *breather*. These are spatially localized, time-periodic dynamical states, which are ubiquitous in nonlinear physical systems ranging from quasi-one dimensional polymers [6] and charge-density-wave materials (e.g., metal-halogen electronic chains [7]) to Josephson ladders [8]. Scattering of a moving solitary pulse on a breather is an important process in many physical, chemical and biological systems which combine charge, spin and energy localization with transport of these quantities by pulses. Detailed understanding of this type of the scattering will not only yield valuable information on the dynamical properties of breathers, such as their mobility and stability, but also reveal the dynamical response of materials in which the breathers are excited. In particular, the interaction of a narrow optical pulse with a pre-existing electron-phonon breather in a lattice chain is a process of considerable fundamental and technological interest, since it may enhance optical nonlinearities of the material, and thus the efficiency of the second- and third-harmonic generation. Indeed, there is some evidence of such an effect in conjugated polymers such as polyenes [9].

Another relevant example of the breathers is provided by the rotational dynamics of certain chemical groups in a molecular chain [10]. Analysis of the pulse scattering on them, in conjunction with recent developments in ultrafast (femtosecond) spectroscopy [11] and chemistry [12], will enable an efficient use of the rich photophysics of functional optical and electronic materials. Damage tracks in certain mica minerals and sputtering on crystal surfaces have also been attributed to moving breathers [13]. In addition to mixed-valence transition-metal complexes [7], there is experimental evidence for localized breather-like states in antiferromagnetic chains [14], as well as in the above-mentioned Josephson junction arrays and ladders [8].

The one-dimensional (1D) model introduced and considered in this work is a first step in a systematic study of the pulse-breather scattering, the eventual objective being to elucidate the interaction of pulses and breathers in 2D and 3D nonlinear dynamical lattices. However, the formulation and consideration of the corresponding multidimensional models is a complicated issue, which is beyond the scope of this work.

The simplest generalization of Eq. (1) for the case when the local nonlinearity is induced by a small-size breather oscillating at a frequency ω is

$$iu_z - \frac{1}{2}\beta_2 u_{\tau\tau} + \frac{i}{6}\beta_3 u_{\tau\tau\tau} + \gamma |u|^2 u + \Gamma \delta(z) \cos^2(\omega t) \cdot |u|^2 u = 0. \quad (3)$$

Equation (3) implies that oscillations of the breather modulate the strength of the corresponding local nonlinear defect, but do not change its sign, which is a natural assumption in the case of the cubic nonlinearity. Note that, despite the difference between the time t and the reduced time τ , see Eq. (2), it makes no difference what definition of time is used in the argument of \cos^2 in Eq. (3), as $\tau \equiv t$ at $z = 0$.

Both equations (1) and (3) conserve the norm of the wave field (which has the physical meaning of energy in the application to fiber optics [2]),

$$E = \int_{-\infty}^{+\infty} |u(\tau)|^2 d\tau. \quad (4)$$

Additionally, the model (1) including the static defect conserves the field momentum,

$$P = i \int_{-\infty}^{+\infty} u_\tau u^* d\tau, \quad (5)$$

the asterisk standing for the complex conjugation.

The simplest and perhaps most fundamental version of both models (1) and (3) is the one with all the nonlinearity concentrated at the point $z = 0$, while the bulk nonlinearity is negligible, i.e., $\gamma = 0$. It appears, rather surprisingly, that, unlike the more technically involved case of the fully nonlinear model with $\gamma > 0$, which was studied in a part in Ref. [1], in the case $\gamma = 0$ the interaction of the moving pulse with the nonlinear defect has not been yet considered. Therefore, the detailed study of the pulse-defect interaction in the model with $\gamma = 0$ is the first objective of the present work. Then, we will also consider the full nonlinear model with $\gamma \neq 0$, concluding that, as a matter of fact, the most interesting results can be obtained just in the model with the linear host medium, $\gamma = 0$.

In the linear medium, it is natural to take an incident pulse in the form of a Gaussian, which is an eigenmode of the linear system. The problem of the interaction of a Gaussian with the delta-like nonlinear defect can be partially solved in an analytical form, which is done below in section 2. In fact, the analytical result can be obtained in an advanced form for the case of the *static* defect with $\omega = 0$, which corresponds to Eq. (1) and, as it was explained above, has direct application to fiber optics. A basic result produced by the analytical consideration is that the initial Gaussian pulse, passing through the nonlinear defect, generates an infinite series of strongly overlapped Gaussian pulses. At this stage of the analysis, TOD becomes a crucially important ingredient of the model: we show that it lends each Gaussian pulses its own velocity, which will bring about eventual separation of the pulses. In section 2, the velocity generated by TOD is found in an analytical form, which yields an exact result for an initial stage of the evolution. The splitting of the solution into an array of pulses is considered in detail in section 3.

In section 4 we display results of the fully numerical solution for the case when the incident Gaussian pulse interacts with the dynamical defect [the breather, see Eq. (3)]. In this case ($\omega \neq 0$), the eventual pattern is much less regular, consisting of separated pulses with random shapes, rather than Gaussian-like ones. However, if the frequency ω is very large, one should expect that the dynamical defect described by the last term in Eq. (3) may be replaced by its averaged static counterpart. In accordance with this expectation, the numerical computations demonstrate return to a regular pattern for very large values of ω .

In section 5 we present a solution for the most general model, which includes the bulk nonlinearity at $z \neq 0$, i.e., $\gamma \neq 0$ in Eq. (3). In this case, it is natural to take the incident pulse as a *soliton* of the corresponding homogeneous NLS equation, rather than a Gaussian pulse, and the interaction may only be simulated numerically. Results of simulations of the interaction of the soliton with the defect turn out to be quite different from those in the case when the host medium was linear: the soliton is found to be much more resistant to the action of the local nonlinear defect than the Gaussian pulse in the linear medium. Unless the defect is extremely strong, the pulse remains virtually intact, the TOD term separating it from small-amplitude wave packets generated by the local defect. However, in the absence of the TOD term, the effect of the perturbation may accumulate and destroy the soliton. Thus, the third-order dispersion plays a crucially important role in *both* versions of the present model, i.e., for the linear and nonlinear host medium.

If the defect is very strong, it may split the soliton into two. The results obtained for the model with the nonlinear host medium are not sensitive to the value of the defect's intrinsic frequency ω , being essentially the same for the static defect and its dynamical counterpart. Moreover, the soliton is found to be stable against the action of the local defect irrespective of the relative sign of the bulk and local nonlinearities. These results, obtained for the case when the input signal is a soliton, rather than a linear pulse, is another manifestation of the well-known general principle, according to which solitons are very robust eigenmodes of nonlinear media.

II. ANALYTICAL CONSIDERATION OF THE INTERACTION BETWEEN A GAUSSIAN PULSE AND A NONLINEAR DEFECT.

A. The pulse

We start the consideration with the simplest version of the model, viz., Eqs. (1) or (3) with $\gamma = \beta_3 = 0$. At $z \neq 0$, we thus have a linear Schrödinger equation, which gives rise to the well-known exact solution in the form of a Gaussian pulse (which is termed a coherent state in quantum mechanics),

$$u(z, \tau) = \frac{A_0}{\sqrt{1 - 2i\beta_2 cz}} \exp \left[-\frac{c(1 + 2i\beta_2 cz)}{1 + (2\beta_2 cz)^2} \cdot \tau^2 \right], \quad (6)$$

where $c > 0$ and A_0 are arbitrary real constants. The constant c determines the width of the Gaussian pulse and its *chirp* (i.e., the imaginary part of the coefficient in front of τ^2 in the argument of the exponential, which shows a slope of the local frequency across the pulse [2]).

Note that the solution (6) has zero velocity in the present reference frame [which is defined by Eq. (2)]. An exact solution for a moving pulse can be generated from Eq. (6) by the action of a *boost* (Galilean transform)

$$u(z, t) \rightarrow u(z, \tau - sz) \exp \left(\frac{is^2}{2\beta_2} z - \frac{is}{\beta_2} \tau \right), \quad (7)$$

where s is a real velocity-shift parameter. It is important to notice that, for the boosted pulse, the values of its velocity, momentum (5) and energy (4) are related, irrespective of the particular form of the pulse, in a simple way,

$$s = \beta_2 \frac{P_{\text{pulse}}}{E_{\text{pulse}}}. \quad (8)$$

B. Pulse acceleration by the third-order dispersion and numerical verification

Before proceeding to the consideration of the passage of the pulse (6) through the nonlinear defect, we need to understand how a free pulse will move under the action of the TOD term in Eqs. (1) or (3). The consideration of this issue will help to understand how an array of pulses splits in the presence of the TOD term. To this end, a solution is sought for by means of the Fourier transform, which yields the following integral representation for it:

$$u(z, \tau) = \frac{A_0}{2\sqrt{\pi c}} \int_{-\infty}^{+\infty} \exp \left[-\frac{\omega^2}{4c} - i\omega\tau + i \left(\frac{1}{2}\beta_2\omega^2 - \frac{1}{6}\beta_3\omega^3 \right) z \right] d\omega \quad (9)$$

[setting $z = 0$, this expression goes over into Eq. (6) taken at $z = 0$]. In the limit of large values of z , the integral (9) is dominated by a contribution from a vicinity of the stationary-phase point, which is

$$\omega_0 \approx \frac{\tau}{\beta_2 z} - \frac{i\tau}{2c\beta_2^2 z^2} + \frac{\beta_3 \tau^2}{2\beta_2^3 z^2}. \quad (10)$$

In particular, a contribution of the stationary-phase point (10) to the phase of the pulse is

$$\phi(z, \tau) = \frac{\tau^2}{2\beta_2 z} - \frac{\beta_3 \tau^3}{6\beta_2^3 z^2}. \quad (11)$$

Direct simulations show that, under the action of TOD, the initial Gaussian pulse is gradually destroyed, generating a long “tail”, while a part of the wave packet may still be interpreted as a surviving pulse. Using the expression (11), one can find a relation between the momentum [see Eq. (5)] and energy of the pulse-like part of the wave packet,

$$P_{\text{pulse}} = \frac{\beta_3 c}{2\beta_2} E_{\text{pulse}}. \quad (12)$$

Note that, as the *net* momentum of the wave field must be conserved, a “recoil” momentum $-P_{\text{pulse}}$ is carried away by the above-mentioned tail. The comparison of Eqs. (8) and (12) yields an analytical prediction for the asymptotic value of the velocity shift acquired by the surviving pulse under the action of the TOD term in the limit $z \rightarrow \infty$:

$$s = (1/2)\beta_3 c. \quad (13)$$

To this end, a natural definition of the position of the wave-packet's center of mass is adopted,

$$\tau_c \equiv E^{-1} \int_{-\infty}^{+\infty} \tau |u(\tau)|^2 d\tau, \quad (14)$$

where E is the net energy defined by Eq. (4), and the derivative $d\tau_c/dz$ may be regarded as a velocity of the pulse.

In the case of the free propagation, one can derive an exact relation

$$\frac{d\tau_c}{dz} = \beta_2 \frac{P}{E} - \frac{1}{2} \beta_3 E^{-1} \int_{-\infty}^{+\infty} |u_\tau|^2 d\tau, \quad (15)$$

where P and E are the net momentum and energy of the wave field. Comparing this to Eq. (8), we conclude that, in the absence of TOD, the velocity (15) is identical to the boost parameter (in particular, $d\tau_c/dz = 0$ if $P = 0$); however, this identity is broken by TOD, that is why acceleration of the pulse by TOD is observed. If the TOD term is treated as a small perturbation, one can analytically calculate the second term on the right-hand side of Eq. (15), using the expression (6), which is an exact solution in the absence of TOD. As a result, we find the velocity of the pulse in the case when the net field momentum vanishes, $P = 0$, which is true for the initial Gaussian pulse (6):

$$\frac{d\tau_c}{dz} = -\frac{1}{3} \beta_3 c \quad (16)$$

[note that this expression does not contain neither z nor β_2 , despite the fact the solution (6), used for the calculation of the right-hand side of Eq. (16), does depend on z and β_2]. The expression (16) shows that the velocity lent to the pulse by TOD linearly depends on the pulse's parameter c , hence initially overlapping pulses with different values of c [see Eq. (21) below] are expected to separate under the action of TOD.

The analytical result (21) was checked against direct numerical simulations of the linear version of Eq. (1). For instance, in the case $\beta_2 = 1$, $\beta_3 = 0.1$, and $c = 1$, it was found that the pulse was indeed moving at a constant velocity, the value of which exactly coincided with that given by Eq. (16), in the interval $0 < z < 25$. At larger values of z , the absolute value of the velocity decreases, which can be explained by the fact that the TOD term essentially alters the shape of the pulse at that later stage of the evolution.

C. Passage of the pulse through the nonlinear defect

Our next aim is to consider transformation of the Gaussian pulse passing the nonlinear defect. Obviously, in an infinitesimal vicinity of the defect (as $|z| \rightarrow 0$), only the first and last terms should be kept in Eqs. (1) and (3), which yields a simplified equation,

$$\frac{\partial u}{\partial z} = i\Gamma \delta(z) \cos^2(\omega\tau) \cdot |u|^2 u. \quad (17)$$

To solve Eq. (17), we represent the solution as $u(z) \equiv a(z) \exp[i\phi(z)]$, with real amplitude a and phase ϕ . Substituting this into Eq. (17), one immediately finds that $\partial a/\partial z = 0$, and

$$\frac{\partial \phi}{\partial z} = \Gamma \delta(z) \cos^2(\omega\tau) \cdot a^2.$$

A solution to the latter equation is obvious,

$$\phi(z = +0, \tau) - \phi(z = -0, \tau) = \Gamma a^2(\tau) \cdot \cos^2(\omega\tau), \quad (18)$$

where we take into regard that a may be a function of τ .

Thus, we take the input pulse at the point $z = -0$ in the general form [cf. the expression (6)],

$$u(z = -0) = A_0 \exp[-(c_0 + ib_0)\tau^2], \quad (19)$$

where $c_0 > 0$ determines the initial width of the pulse, and b_0 is its initial chirp. The substitution of the expression (19) into Eq. (18) yields the form of the pulse appearing after the passage of the nonlinear defect:

$$u(z = +0, \tau) = A_0 \exp[-(c_0 + ib_0)\tau^2 + i\Gamma A_0^2 \exp(-2c_0\tau^2) \cdot \cos^2(\omega\tau)]. \quad (20)$$

Further analytical consideration for the general case of the dynamical defect (breather), with $\omega \neq 0$, is extremely cumbersome. Therefore, in the rest of this section and in the next one, we focus on the static case, $\omega = 0$. The aim will be to realize a result of the further evolution of the transformed pulse (20), governed by the linear equation (1) Including the TOD term) at $z > 0$. To this end, we notice that the expression (20) can be expanded into an infinite series:

$$u(z = +0) = A_0 \exp(-ib_0\tau^2) \sum_{n=0}^{+\infty} (n!)^{-1} (i\Gamma A_0^2)^n \exp[-(1+2n)c_0\tau^2]. \quad (21)$$

Comparing it to the exact fundamental-pulse solution (6), we conclude that the expression (21), if considered as an initial condition to the linear equation (1) with $\gamma = \beta_3 = 0$, gives rise to a superposition of an infinite number of Gaussians with the values of the width constant $c_n = (1+2n)c_0$, and with the common initial value b_0 of the chirp. The evolution of the wave packet (21) can be presented in a relatively simple form in the case when the initial chirp is absent, $b_0 = 0$ (and TOD is neglected, $\beta_3 = 0$):

$$u(z, \tau) = A_0 \exp(-ib_0\tau^2) \sum_{n=0}^{+\infty} \frac{(i\Gamma A_0^2)^n}{n! \sqrt{1 - 2i\beta_2 c_0 (1+2n)z}} \exp\left[-\frac{c_0(1+2i\beta_2 c_0(1+2n)z)}{1 + (1+2n)^2 (2\beta_2 c_0 z)^2} \tau^2\right] \quad (22)$$

[in the case $\omega \neq 0$, Eq. (20) shows that the Gaussian in each term of the initial series (21) with $n \neq 0$ is additionally multiplied by $[\cos(\omega t)]^{2n}$, which will make the subsequent result much more complex than that given by Eq. (22)].

Thus, Eq. (22) gives an exact solution to the nonlinear model equation (1) in the case when the incident pulse has no chirp and $\gamma = \beta_3 = 0$. However, while taking the input pulse to be chirpless, and disregarding the bulk nonlinearity ($\gamma = 0$) are quite acceptable assumptions, the TOD term may *not* be neglected, as, without this term, the exact solution (22) remains strongly degenerate. Indeed, centers of all the Gaussian pulses, the superposition of which constitutes this solution, exactly coincide, staying at $\tau = 0$ [note that this degeneracy is not lifted if the input pulse has nonzero chirp, nor if the bulk nonlinearity ($\gamma \neq 0$) is added]. On the other hand, Eq. (16) shows that the TOD term will lend each pulse its own velocity, depending on the initial width parameter c of the pulse. Obviously, this will eventually split the superposition (22) of the overlapping Gaussians into an array of separating pulses.

However, the analytical prediction (16) for the TOD-induced velocity shift of each pulse is only valid for finite values of z . Therefore, to find an actual shape of the evolving wave train, it is necessary to solve numerically the linear Schrödinger equation with the TOD term,

$$iu_z - \frac{1}{2}\beta_2 u_{\tau\tau} + \frac{i}{6}\beta_3 u_{\tau\tau\tau} = 0, \quad (23)$$

with the initial condition in the form (20). Results generated by Eq. (23) are presented in the next section.

III. GENERATION OF THE WAVE TRAIN BY THE THIRD-ORDER DISPERSION IN THE CASE OF THE STATIC NONLINEAR DEFECT

As Eq. (23) is linear and has constant coefficients, it can be solved by Fourier transform. Thus, the numerical part of the solution amounts to the computation of the Fourier transform

$$u_0(\omega) \equiv \int_{-\infty}^{+\infty} \exp(i\omega\tau) u(z = +0, \tau) d\tau$$

for the initial configuration (20), and subsequent computation of the inverse Fourier transform

$$u(z, \tau) \equiv (2\pi)^{-1} \int_{-\infty}^{+\infty} \exp(-i\omega\tau) u(z, \omega) d\omega,$$

where, as it immediately follows from Eq. (23),

$$u(z, \omega) = u_0(\omega) \exp\left[i\left(\frac{1}{2}\beta_2\omega^2 z - \frac{1}{6}\beta_3\omega^3 z\right)\right].$$

In Fig. 1 we display the profiles of $|u(\tau)|$, computed at the points $z = 10, 20, 30$, and 40 for a case when the static nonlinear defect is weak, having $\Gamma = 0.1$. The figure demonstrates that, in accordance with the analysis presented in

the previous section, the initial wave packet tends to split into an array of regular Gaussian-like pulses. The splitting actually takes place for larger values of the nonlinear-defect's strength, as is shown in Fig. 2, which pertains to $\Gamma = 1$. For the same case, the eventual shape of the pulse array is shown in more detail in Fig. 3.

The result for a still stronger nonlinear defect, with $\Gamma = 10$, is displayed in Fig. 4. In this case, the splitting into pulses takes a violent character, with appearance of huge gradients and formation of a very sharp front, the latter effect being accounted for by the interplay of the strong local nonlinearity and TOD. These results may be explained by the fact that, as follows from Eq. (21), the number n_{\max} of the largest-amplitude pulse in the series grows $\sim \Gamma$ with increase of Γ , hence the width W of an individual pulse decreases $\sim 1/\sqrt{\Gamma}$, which gives rise to the large gradients revealed by the subsequent evolution. We also note that, although the TOD coefficient β_3 remains small, the size of the TOD term in Eqs. (1) and (3) grows as $1/W^3$ with the decrease of the pulse's width. This explains the formation of the abrupt front under the action of the asymmetric TOD term.

IV. GENERATION OF THE PULSE ARRAY BY A LOCALIZED BREATHER

An example of the splitting of the initial Gaussian as a result of its interaction with the dynamical defect (with the same strength $\Gamma = 1$ as in the case shown in Fig. 2, and the frequency $\omega = 1/2$) is shown in Fig. 5. The splitting is displayed in more detail by blowups collected in Fig. 6.

Comparison of Figs. 2 and 5 shows that the static defect and the dynamical one with a moderate value of the frequency produce quite similar results. Taking larger values of the frequency strongly changes the situation: as is shown in Fig. 7 and in the accompanying blowup (Fig. 8), the same value of the defect strength as in the cases displayed in Figs. 2 and 5, i.e., $\Gamma = 1$, but combined with $\omega = 10$, gives rise to an essentially more disordered pattern. Note that this pattern is disordered in a way essentially different from that observed as a result of the action of a strong static defect ($\omega = 0$), cf. Fig. 4. In particular, strong asymmetry of the pattern and sharp fronts are not found in the present case, and the local gradients are not as huge as in Fig. 4. The absence of those features in the present case is easy to understand, as they may only be generated by a large value of Γ , as explained above.

Keeping to increase ω at a fixed value of Γ , we have concluded that the situation shown in Figs. 7 and 8 is rather similar to that observed at $\Gamma = 1$ and $\omega = 100$ (not shown here). On the other hand, it is obvious that, if the frequency is extremely large, one should be able to replace $\cos^2(\omega t)$ in Eq. (3) by its average value $1/2$, thus reverting to Eq. (1) for the static nonlinear defect. To verify this argument, in Fig. 9 we present the results of the numerical computations for $\Gamma = 1$ and $\omega = 1000$, which are supported by the blowup shown in Fig. 10. From these pictures, it is evident that, in the case of extremely large ω , the situation is indeed nearly the same as in the case of the static nonlinear defect, cf. Figs. 2 and 3.

V. TRANSFORMATION OF A SOLITON BY THE LOCAL DEFECT IN THE NONLINEAR HOST MEDIUM

In all the cases considered above, the input pulse was taken in the form of a Gaussian, since the host medium in which this pulse propagated before the collision with the nonlinear defect was linear, the Gaussian being its eigenmode. The situation is completely different in the case when the host waveguide is itself nonlinear, as in that case the incoming pulse must be a sech soliton [1]. The action of the localized nonlinearity on the soliton is described by the same general expression (18) as above; however, the result cannot be interpreted in such a straightforward manner (even if $\omega = 0$) as it was done above for the case of the input Gaussian pulse, see Eq. (21). In fact, in the case of the nonlinear host medium all the analysis following the use of expression (18) for the defect-induced phase change of the input soliton can only be performed numerically.

As is commonly known, in the homogeneous part of the nonlinear model (1) or (3) (at $z \neq 0$), the soliton may only exist if the second-order dispersion is anomalous while the nonlinearity is self-focusing, or vice versa [2], i.e., only if $\beta_2\gamma < 0$. In the simulations, we fixed $\beta_2 \equiv +1$, and took different negative values of γ , see below. The input soliton was always taken as a fundamental one with a fixed width corresponding to these values of the parameters:

$$u(z = -0, \tau) = |\gamma|^{-1/2} \text{sech } \tau. \quad (24)$$

If the TOD term is neglected ($\beta_3 = 0$), the nonlinear defect produces a strong perturbation around the soliton. However, the TOD term helps to separate the soliton and the perturbation, as is illustrated by Fig. 11 for the case $\beta_3 = 0.3$. This figure shows a large τ -domain, in order to demonstrate the evolution over a large propagation distance; as is seen, the perturbation spreads out indefinitely, while the soliton remains essentially intact.

The comparison of Fig. 11 with Fig. 2, which shows the interaction of the Gaussian pulse in the linear host medium with the local nonlinear defect that has the same strength, $\Gamma = 1$, suggests a conclusion that the soliton in the nonlinear host medium is much more resistant to the action of the nonlinear defect than the Gaussian pulse in the linear host medium. In fact, this conclusion is strongly supported by many other simulations with different values of the parameters.

Another difference of the present case from that for the Gaussian pulse in the linear host medium is that the result of the action of the nonlinear defect on the soliton is less sensitive to the defect's intrinsic frequency. For instance, if the defect is dynamical with $\omega = 1$, while the other parameters take the same values as in the case shown in Fig. 11, the evolution (not shown here) seems nearly the same as in Fig. 11, with a difference that the generated perturbation takes a somewhat larger portion of energy from the soliton (as a result, the soliton reappears after the collision with the amplitude ≈ 0.8 , to be compared with the amplitude ≈ 1 in Fig. 11). Nevertheless, in this case too, the perturbation remains rather small and it does not strongly affect the soliton.

With further increase of the dynamical defect's frequency, the perturbation again becomes smaller, and the general picture is reverting to that corresponding to the static defect with $\omega = 0$. This trend can be easily explained by the self-averaging of the dynamical defect similar to that observed above in the case of the linear host medium. However, unlike that case, where the self-averaging was evident only for extremely large values of the frequency, $\omega \sim 1000$ (see Fig. 9), in the present case simulations (not shown here) clearly show that $\omega = 10$ is sufficient for the self-averaging to manifest itself.

The above results were obtained for the cases with $\Gamma > 0$ and $\gamma < 0$, which implies that the localized nonlinearity is self-defocusing if the bulk nonlinearity is self-focusing, or vice versa. It is also interesting to consider the case when both nonlinearities have the same sign, i.e., $\Gamma < 0$. An example is displayed in Fig. 12 for $\Gamma = -1$ (this example pertains to the dynamical defect with $\omega = 1$, but the results are virtually the same for the static defect with $\omega = 0$). In this case, the action of the defect produces a weaker effect, as the soliton reappears after the interaction with the amplitude ≈ 1 , to be compared with the above-mentioned value ≈ 0.8 of the soliton's amplitude found for the same values of parameters but $\Gamma = +1$. This difference is quite natural, as the local defect which has the same sign of the nonlinearity as in the host medium is acting to compress the soliton stronger, while the local defect with the opposite sign of Γ was acting to stretch the soliton, which produces a more destructive effect, helping some radiation (wave packets) to escape from the weakened soliton pulse.

Taking a much stronger nonlinear defect (with $\Gamma = 11$, see Fig. 13), we conclude that, naturally, the soliton loses a larger part of its energy to the generation of the perturbation separating from it. Nevertheless, the soliton survives even in this case (cf. Fig. 7, which shows a complete and fast chaotization of the wave field in the case of the collision of the Gaussian pulse with a strong defect, having $\Gamma = 10$, in the linear host medium). In the cases when Γ is large but negative (not shown here), when the nonlinearity of the defect has the same sign as in the host medium, the soliton's losses are smaller than in the case displayed in Fig. 13, which is quite similar to what is shown in Fig. 12 for a moderately strong defect with $\Gamma < 0$.

VI. CONCLUSION

We have introduced a model describing collision of a pulse in a linear or nonlinear waveguide with a strong nonlinear local defect, that may be either static or breather-like. The model with the static defect should directly apply to an optical pulse in a long fiber-optic link with an inserted section of a nonlinear (dispersion-shifted) fiber of an arbitrary length. On the other hand, a local breather, which gives rise to the nonlinear defect affecting the propagation of narrow optical pulses, may be realized in a molecular chain with electron-phonon coupling.

In the case when the host waveguide is linear, the pulse was naturally taken as a Gaussian. A result of its interaction with the nonlinear defect was found analytically, amounting to its transformation into a "lump" consisting of an infinite number of overlapping Gaussian pulses. Further evolution of the lump in the linear medium is generated by the corresponding Fourier transform. An important ingredient of the medium is the third-order dispersion, that splits the lump into an array of individual pulses; the velocity shift lent to an initially Gaussian pulse by the TOD term was found in an analytical form. The influence of the intrinsic frequency, in the case when the defect is a breather, was also investigated.

The full numerical solution for the model in which the host medium is nonlinear, and the input pulse is taken as a fundamental soliton shaped by this medium, produces a result quite different from that for the linear host medium: the soliton is much more resistant to the action of the local nonlinearity. If the local defect is not very strong, the soliton remains essentially intact, the third-order-dispersion separating the soliton and small wave packets generated by the collision. Even if the local nonlinearity is very strong, the soliton survives, losing a limited part of its energy.

Beyond straightforward application of the results reported above to fiber-optic links, the model introduced in this

work, and current developments in experimental techniques, such as ultrafast spectroscopy [11,12] of optical and electronic materials [7], the results presented here may help to understand the dynamics of scattering of pulses on static and breather-like defects in many physical systems, e.g., conjugated polymers [9,3–5], Josephson ladders [8], and coupled electron-vibron lattice systems [15].

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FIGURE CAPTIONS

Fig. 1. The evolution of the field $|u(\tau)|$ after the interaction of the input Gaussian pulse with the nonlinear defect, in the case $\omega = \gamma = 0$, $\beta_2 = 1$, $\beta_3 = 0.1$, and $\Gamma = 0.1$.

Fig. 2. The same as in Fig. 1 for a stronger nonlinear defect, with $\Gamma = 1$.

Fig. 3. Blowups of the field pattern from the last panel in Fig. 2, clearly showing the formation of an array of separated pulses.

Fig. 4. The same as in Fig. 3 for a very strong static nonlinear defect, with $\Gamma = 10$.

Fig. 5. The evolution of the field $|u(\tau)|$ after the interaction of the input Gaussian with the nonlinear dynamical defect, in the case $\omega = 1/2$, $\gamma = 0$, $\beta_2 = 1$, $\beta_3 = 0.1$, and $\Gamma = 1$. Except for ω , these parameters are the same as in the case shown in Fig. 2.

Fig. 6. Blowups of segments of the panels from Fig. 5, showing the gradual splitting of the wave packet into an array of pulses.

Fig. 7. The same as in Fig. 5, but with $\omega = 10$.

Fig. 8. The same as in Fig. 6, but with $\omega = 10$.

Fig. 9. The same as in Fig. 7, but with $\omega = 1000$.

Fig. 10. The same as in Fig. 6, but with $\omega = 1000$.

Fig. 11. The evolution of the field $|u(\tau)|$ over a very large propagation distance in a large temporal domain after the interaction of the input soliton (24) with the nonlinear static defect, in the case $\gamma = -\beta_2 = -1$, $\beta_3 = 0.3$, and $\Gamma = 1$, $\omega = 0$. It is evident that the local defect initially creates a large perturbation around the soliton; however, the perturbation is diffused away under the combined action of the second- and third-order dispersions, and a robust soliton reappears.

Fig. 12. The same as in Fig. 11, but with $\omega = 1$ and $\Gamma = -1$.

Fig. 13. The same as in Fig. 12, but with $\beta_3 = 0.5$ and $\Gamma = +11$.